

# Soft $\Gamma$ -Semirings

Ö. Bektaş, N. Bayrak, B. A. Ersoy

obektas@yildiz.edu.tr, nbayrak@yildiz.edu.tr, ersoya@yildiz.edu.tr

Yildiz Technical University, Faculty of Arts and Sciences

Department of Mathematics, Istanbul, TURKEY

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## Abstract

In this paper, the definitions of soft  $\Gamma$ -semirings and soft sub  $\Gamma$ -semirings are introduced with the aid of the concept of soft set theory introduced by Molodtsov. In the mean time, some of their properties and structural characteristics are investigated and discussed. Thereafter, several illustrative examples are given.

**Keywords:** Soft Sets, Fuzzy Sets, Semirings,  $\Gamma$ -ring.

## 1 Introduction

Uncertain data modelling was investigated by many researchers in economics, engineering, environmental sciences, sociology, medical science and many other fields. The process in classical mathematics may not be competent owing to the fact that the assorted uncertainties deriving in these fields. In this context, mathematical theories such as probability theory, fuzzy set theory [1], rough set theory [2] were established by researchers to modelling uncertainties arising in the stated fields. In 1999, Molodtsov [3] made a new viewpoint of substantial theoretical approaches: the concept of soft set theory which is more convenient than classical ideologies and can be seen as a outstanding mathematical tool relates with uncertainties. After Molodtsov's work, some different applications of soft sets were studied in [4, 5, 6, 7].

The algebraic structure of soft set theories has been studied progressively in recent years. Aktaş and Çağman [8] investigated basic properties of soft sets to the related concepts of fuzzy sets and rough sets. They also defined the notion of soft groups, and derived some related properties. Furthermore, Maji et al. [9, 10] presented the definition of fuzzy soft set. The concept of fuzzy soft groups which is a generalization of soft groups were given in [11] and [12]. In 2010, a tentative approach between fuzzy sets (rough sets) and soft sets were studied by Feng et al. in [13].

On the other hand soft rings, soft ideals on soft rings and idealistic soft rings were defined in [14]. After these studies the notion of fuzzy soft rings and

fuzzy soft ideals were discussed in [15]. In addition to this in [16] the concept of soft BCH-algebra was introduced and some of their properties and structural characteristics were mentioned.

Furthermore the notion of soft semirings are investigated in [17] which is useful for dealing with problems in different areas of applied mathematics and information sciences. The semiring structure provides an algebraic framework for modelling and investigating the key factors in these problems. Then, N. Nobusawa [18] introduced the notion of  $\Gamma$ -ring, as more general than ring. After that, the weakened conditions of the definition of the  $\Gamma$ -ring were studied in [19]. Then the generalization of  $\Gamma$ -ring and  $\Gamma$ -semiring were introduced by [20].

Thereafter, in [21] Jun and Lee studied the concept of fuzzy  $\Gamma$ -ring and [22] defined the soft  $\Gamma$ -rings and idealistic soft  $\Gamma$ -rings with their basic properties. The extension of the  $\Gamma$ -semiring to quasi ideals was done by [23, 24, 25] with incompatible style.

In this paper, we introduce the concept of soft  $\Gamma$ -semiring which extend the notion of soft  $\Gamma$ -ring theory and deal with some of its algebraic properties by giving several examples.

## 2 Soft $\Gamma$ -Semirings

**Definition 2.1** A pair  $(\rho, W)$  is called a soft set over  $V$ , where  $\rho$  is a mapping from  $W$  to  $P(V)$  [3].

**Definition 2.2** Let  $(\rho, W), (\sigma, Y)$  be soft sets over a common universe  $V$ .

- i) If  $W \subseteq Y$  and  $\rho(\omega) \subseteq \sigma(\omega)$  for all  $\omega \in W$  then we say that  $(\rho, W)$  is a soft subset of  $(\sigma, Y)$ , denoted by  $(\rho, W) \subseteq (\sigma, Y)$ .
- ii) If  $(\rho, W)$  is a soft subset of  $(\sigma, Y)$  and  $(\sigma, Y)$  is a soft subset of  $(\rho, W)$ , then we say that  $(\rho, W)$  is soft equal to  $(\sigma, Y)$ , denoted by  $(\rho, W) = (\sigma, Y)$ .

**Definition 2.3** i) Let  $(\rho, W)$  and  $(\sigma, Y)$  be two soft set over a common universe  $V$ .

$$(\psi, Z) = (\rho, W) \widetilde{\cap}_{\Re} (\sigma, Y)$$

is said to be restricted-intersection of  $(\rho, W)$  and  $(\sigma, Y)$ , where  $(\psi, Z)$  is soft set,  $Z = W \cap Y \neq \emptyset$  and the mapping  $\psi$  is defined by

$$\begin{aligned} \psi : Z &\rightarrow P(V) \\ z &\rightarrow \psi(z) = \rho(z) \cap \sigma(z) \end{aligned}$$

- ii) Let  $\{(\rho_i, W_i) : i \in I\}$  be non-empty family soft sets. The restricted-intersection of a non-empty family soft sets is defined by

$$(\psi, Y) = (\widetilde{\cap}_{\mathfrak{R}})_{i \in I} (\rho_i, W_i)$$

where  $(\psi, Y)$  is a soft set,  $Y = \bigcap_{i \in I} W_i \neq \emptyset$  and  $\psi(y) = \bigcap_{i \in I} \rho_i(y)$  for every  $y \in Y$  [17],[7].

**Definition 2.4** i) Let  $(\rho, W)$  and  $(\sigma, Y)$  be two soft set over a common universe  $V$ .

$$(\psi, Z) = (\rho, W) \widetilde{\cap}_{\mathcal{E}} (\sigma, Y)$$

is called extended-intersection of  $(\rho, W)$  and  $(\sigma, Y)$ , where  $(\psi, Z)$  is soft set and  $(\psi, Z)$  satisfying the following conditions

- $Z = W \cup Y$
- $\psi(z) = \begin{cases} \rho(z) & , \text{if } z \in W \setminus Y \\ \sigma(z) & , \text{if } z \in Y \setminus W \\ \rho(z) \cap \sigma(z) & , \text{if } z \in c \in W \cap Y. \end{cases}$

ii) Let  $\{(\rho_i, W_i) : i \in I\}$  be non-empty family soft sets. The extended-intersection of a non-empty family soft sets is defined by

$$(\psi, Y) = (\widetilde{\cap}_{\mathcal{E}})_{i \in I} (\rho_i, W_i)$$

where  $(\psi, Y)$  is a soft set,  $Y = \bigcup_{i \in I} W_i$ ,  $\psi(y) = \bigcap_{i \in I} \rho_i(y)$  and  $I(y) = \{i : i \in W_i\}$  for every  $y \in Y$  [7],[17].

**Definition 2.5** i) Let  $(\rho, W)$  and  $(\sigma, Y)$  be two soft set over a common universe  $V$ .

$$(\psi, Z) = (\rho, W) \widetilde{\cup}_{\mathfrak{R}} (\sigma, Y)$$

is said to be restricted union of  $(\rho, W)$  and  $(\sigma, Y)$ , where  $(\psi, Z)$  is soft set,  $Z = W \cap Y \neq \emptyset$ , and the mapping  $\psi$  is defined by

$$\begin{aligned} \psi : Z &\rightarrow P(V) \\ z &\rightarrow \psi(z) = \rho(z) \cup \sigma(z) \end{aligned}$$

[7].

ii) Let  $\{(\rho_i, W_i) : i \in I\}$  be non-empty family soft sets. The restricted-union of a non-empty family soft sets is defined by

$$(\psi, Y) = (\widetilde{\cup}_{\mathfrak{R}})_{i \in I} (\rho_i, W_i)$$

where  $(\psi, Y)$  is a soft set,  $Y = \bigcap_{i \in I} W_i \neq \emptyset$  and  $\psi(y) = \bigcup_{i \in I} \rho_i(y)$  for every  $y \in Y$  [16].

**Definition 2.6** i) Let  $(\rho, W)$  and  $(\sigma, Y)$  be two soft set over a common universe  $V$ .

$$(\psi, Z) = (\rho, W) \widetilde{\cup}_{\mathcal{E}} (\sigma, Y)$$

is said to be extended union of  $(\rho, W)$  and  $(\sigma, Y)$  where  $(\psi, Z)$  is a soft set,  $Z = W \cup Y$ , and the mapping  $\psi$  is defined by

$$\psi(z) = \begin{cases} \rho(z) & , \text{if } z \in W \setminus Y \\ \sigma(z) & , \text{if } z \in Y \setminus W \\ \rho(z) \cup \sigma(z) & , \text{if } z \in c \in W \cap Y. \end{cases}$$

ii) Let  $\{(\rho_i, W_i) : i \in I\}$  be non-empty family soft sets. The extended-union of a non-empty family soft sets is defined by

$$(\psi, Y) = (\widetilde{\cup}_{\mathcal{E}})_{i \in I} (\rho_i, W_i)$$

where  $(\psi, Y)$  is a soft set,  $Y = \bigcup_{i \in I} W_i$ ,  $\psi(y) = \bigcup_{i \in I} \rho_i(y)$  and  $I(y) = \{i : i \in W_i\}$  for every  $y \in Y$  [7].

**Definition 2.7** i) Let  $(\rho, W)$  and  $(\sigma, Y)$  be two soft set over a common universe  $V$ .

$$(\psi, Z) = (\rho, W) \widetilde{\Lambda} (\sigma, Y)$$

is called  $\Lambda$ -intersection of  $(\rho, W)$  and  $(\sigma, Y)$ , where  $(\psi, Z)$  is soft set,  $Z = W \times Y$  and  $\psi(w, y) = \rho(w) \cap \sigma(y)$  for every  $(w, y) \in W \times Y$ .

ii) Let  $\{(\rho_i, W_i) : i \in I\}$  be non-empty family soft sets. The  $\Lambda$ -intersection of a non-empty family soft sets is defined by

$$(\psi, Y) = \widetilde{\Lambda}_{i \in I} (\rho_i, W_i)$$

where  $(\psi, Y)$  is a soft set,  $Y = \prod_{i \in I} W_i$  and  $\psi(y) = \bigcap_{i \in I} \rho_i(y)$  for every  $y = (y_i)_{i \in I} \in Y$  [6, 17].

**Definition 2.8** i) Let  $(\rho, W)$  and  $(\sigma, Y)$  be two soft set over a common universe  $V$ .

$$(\psi, Z) = (\rho, W) \widetilde{\vee} (\sigma, Y)$$

is called  $\vee$ -union of  $(\rho, W)$  and  $(\sigma, Y)$ , where  $(\psi, Z)$  is soft set,  $Z = W \times Y$  and  $\psi(w, y) = \rho(w) \cup \sigma(y)$  for every  $(w, y) \in W \times Y$ .

ii) Let  $\{(\rho_i, W_i : i \in I)\}$  be non-empty family soft sets. The  $\vee$ -union of a non-empty family soft sets is defined by

$$(\psi, Y) = \widetilde{\vee}_{i \in I} (\rho_i, W_i)$$

where  $(\psi, Y)$  is a soft set,  $Y = \prod_{i \in I} W_i$  and  $\psi(y) = \bigcup_{i \in I} \rho_i(y)$  for every  $y = (y_i)_{i \in I} \in Y$  [6, 17].

**Definition 2.9** *i) Let  $(\rho, W)$  and  $(\sigma, Y)$  be two soft sets over a common universe  $V_1$  and  $V_2$  respectively. The cartesian product of two soft sets  $(\rho, W)$  and  $(\sigma, Y)$  is defined by*

$$(Z, W \times Y) = (\rho, W) \times (\sigma, Y)$$

*where  $(Z, W \times Y)$  is a soft set, and  $\psi(\omega, y) = \rho(\omega) \times \sigma(y)$  for every  $(w, y) \in W \times Y$  [6].*

*ii) Let  $\{(\rho_i, W_i) : i \in I\}$  be non-empty family soft sets over  $V_i, i \in I$ . The cartesian product of a non-empty family soft sets  $\{(\rho_i, W_i) : i \in I\}$  over the universes  $V_i$ , is defined by*

$$(\psi, Y) = \widetilde{\Pi}_{i \in I} (\rho_i, W_i)$$

*where  $(\psi, Y)$  is a soft set,  $Y = \Pi_{i \in I} W_i$  and  $\psi(y) = \Pi_{i \in I} \rho_i(y)$  for all  $y = (y_i)_{i \in I} \in Y$  [16].*

**Definition 2.10** *i) Let  $(\rho, W)$  be soft set over a common universe  $V$ . Then  $(\rho, W)$  is said to be a relative null soft set, denoted by  $N_W$ , if  $\rho(e) = \emptyset$  for every  $e \in W$ .*

*ii)  $(\rho, W)$  is said to be relative whole soft, denoted by  $\mathcal{W}_W$ , if  $\rho(e) = V$  for every  $e \in W$  [16].*

**Definition 2.11** *Let  $(\rho, W)$  and  $(\sigma, Y)$  be two softs set over a common universe  $V_1$  and  $V_2$ , respectively, and  $f : V_1 \rightarrow V_2, g : W \rightarrow Y$  be two functions.  $(f, g)$  is said to be a soft function from  $(\rho, W)$  to  $(\sigma, Y)$ , denoted by  $(f, g) : (\rho, W) \rightarrow (\sigma, Y)$  if the following condition*

$$f(\rho(\omega)) = \sigma(g(\omega))$$

*satisfies for all  $w \in W$ . If  $f$  and  $g$  are injective (resp. surjective, bijective), then we say that  $(f, g)$  is injective(resp. surjective, bijective)[16].*

**Lemma 2.12** *Let  $(\rho, W), (\sigma, Y)$  and  $(\psi, Z)$  be soft sets over  $V_1, V_2$  and  $V_3$ , respectively. If*

$$(f, g) : (\rho, W) \rightarrow (\sigma, Y)$$

*and*

$$(f', g') : (\sigma, Y) \rightarrow (\psi, Z)$$

*are two soft functions, then*

$$(f' \circ f, g' \circ g) : (\rho, W) \rightarrow (\psi, Z)$$

*is a soft function.*

**Definition 2.13** Let  $(\rho, W)$  and  $(\sigma, Y)$  be two soft sets over  $V_1$  and  $V_2$ , respectively,  $(f, g)$  is a soft function from  $(\rho, W)$  to  $(\sigma, Y)$ . The image of  $(\rho, W)$  under the soft function  $(f, g)$ , denoted by  $(f, g)(\rho, W) = (f(\rho), Y)$ , is the soft set over  $V_1$  defined by

$$f(\rho)(y) = \begin{cases} V_{1g(\omega)=y} f(\rho(\omega)) & \text{if } y \in \text{Img} \\ \emptyset & \text{otherwise} \end{cases}$$

for all  $y \in Y$ . The pre-image of  $(\sigma, Y)$  under the soft function  $(f, g)$  denoted by  $(f, g)^{-1}(\sigma, Y) = (f^{-1}(\sigma), W)$ , is the soft set over  $V_1$  defined by  $f^{-1}(\sigma)(\omega) = f^{-1}(\sigma(\rho(\omega)))$  for all  $\omega \in W$ .

It is clear that  $(f, g)(\rho, W)$  is a soft subset of  $(\sigma, Y)$  and  $(\rho, W)$  is a soft subset of  $(f, g)^{-1}(\sigma, Y)$ . In particular, if  $\rho$  is the identity function on  $W$ , the soft sets  $(f(\rho), W)$  and  $f^{-1}(\sigma)(\omega)$  are as given in [8] and [16].

**Definition 2.14** Let  $S$  and  $\Gamma$  be two additive commutative semigroups. Then  $S$  is called a  $\Gamma$ -semiring if there exists a mapping  $S \times \Gamma \times S$  (images to be denoted by  $a\alpha b$  for all  $a, b \in S$  and  $\alpha \in \Gamma$ ) satisfying the following conditions

- i)  $(a + b)\alpha c = a\alpha c + b\alpha c$
- ii)  $a\alpha(b + c) = a\alpha b + a\alpha c$
- iii)  $a(\alpha + \beta)b = a\alpha b + a\beta b$
- iv)  $a\alpha(b\beta c)b = (a\alpha b)\beta c$  for all  $a, b, c \in S$  and for all  $\alpha, \beta \in \Gamma$  [24].

**Example 2.15** Let  $\mathbb{Q}$  be set of rational numbers.  $(S, +)$  be the commutative semigroup of all  $2 \times 3$  matrices over  $\mathbb{Q}$  and  $(\Gamma, +)$  be commutative semigroup of all  $3 \times 2$  matrices over  $\mathbb{Q}$ . Define  $W\alpha Y$  usual matrix product of  $W, \alpha$  and  $Y$ ; for all  $W, Y \in S$  and for all  $\alpha \in \Gamma$ . Then  $S$  is a  $\Gamma$ -semiring but not a semiring [24].

**Remark 2.16** Let  $\mathbb{N}$  be the set of natural numbers and  $\Gamma = \{1, 2, 3\}$ . Define the mapping  $\mathbb{N} \times \Gamma \times \mathbb{N} \rightarrow \mathbb{N}$  by  $a\alpha b = a \cdot \alpha \cdot b$  (usual product of  $a, \alpha$  and  $b$ ); for all  $a, b \in \mathbb{N}, \alpha \in \Gamma$ . Then  $\mathbb{N}$  is a  $\Gamma$ -semiring given in [25]. But  $\Gamma$  is not an additive semigroup, hence it is not a  $\Gamma$ -semiring according to [23].

**Example 2.17** Let  $\mathbb{N}$  be the set of natural numbers and  $\Gamma = \{1, 2, 3\}(\mathbb{N}, \max)$  and  $(\Gamma, \max)$  are commutative semigroups. Define the mapping  $\mathbb{N} \times \Gamma \times \mathbb{N} \rightarrow \mathbb{N}$ , by  $a\alpha b = \min\{a, \alpha, b\}$ , for all  $a, b \in \mathbb{N}, \alpha \in \Gamma$ . Then  $\mathbb{N}$  is a  $\Gamma$ -semiring [24].

**Example 2.18** Let  $\mathbb{Q}$  be set of rational numbers and  $\Gamma = \mathbb{N}$  be the set of natural numbers  $(\mathbb{Q}, +)$  and  $(\mathbb{N}, +)$  are commutative semigroups. Define the mapping  $\mathbb{Q} \times \mathbb{N} \times \mathbb{Q} \rightarrow \mathbb{Q}$  by  $a\alpha b$  usual product of  $a, \alpha, b$ ;  $a, b \in \mathbb{Q}, \alpha \in \Gamma$ . Then  $\mathbb{Q}$  is a  $\Gamma$ -semiring [24].

### 3 Soft $\Gamma$ -Semiring

Let  $S$  be a nonempty set and a  $\Gamma$ -semiring.  $R$  will allude to any triplet relation the midst of a component of  $S$  and a component of  $\Gamma$  and a component of  $S$ , that is, esoterically  $R$  is a subset of  $S \times \Gamma \times S$ . In this way, a set valued function  $\psi : N \rightarrow P(S)$  can be defined as

$$\psi(y) = \{s \in S : R(y, \alpha, s), \forall \alpha \in \Gamma\}$$

for all  $y \in N$ . The pair  $(\psi, N)$  is then a soft set over  $S$ , which produced from the relation  $R$ . The set

$$Supp(\psi, N, \Gamma) = \{y \in N : \psi(y) \neq \emptyset\}$$

is called a support of the soft set  $(\psi, N, \Gamma)$ . The soft set  $(\psi, N, \Gamma)$  non-null if  $Supp(\psi, N, \Gamma) \neq \emptyset$  [16, 17].

**Definition 3.1** A nonempty subset  $T$  of  $S$  is said to be a sub- $\Gamma$ - semiring of  $S$  if  $(T, +)$  is a subsemigroup of  $(S, +)$  and  $a\alpha b \in T$ ; for all  $a, b \in T$  and for all  $\alpha \in \Gamma$  [24].

**Definition 3.2** Let  $(\psi, N)$  be a non-null soft set over a  $\Gamma$ -semiring  $S$ . Then  $(\psi, N)$  is called a soft  $\Gamma$ - semiring over  $S$  if  $\psi(x)$  is a sub- $\Gamma$ -semiring of  $S$  for all  $y \in Supp(\psi, N)$ . This denoted by  $(\psi, N, \Gamma)$ .

**Example 3.3** For consider the additively abelian groups  $\mathbb{Z}_8 = \{0, 1, 2, 3, 4, 5, 6, 7\}$  and  $\Gamma = \{2, 4, 6\}$ . Let  $\cdot : \mathbb{Z}_8 \times \Gamma \times \mathbb{Z}_8 \rightarrow \mathbb{Z}_8, (y, \alpha, s) = y\alpha s$ . Therefore we have that

$$i) (a + b)\alpha c = a\alpha c + b\alpha c$$

$$ii) a\alpha(b + c) = a\alpha b + a\alpha c$$

$$iii) a(\alpha + \beta)b = a\alpha b + a\beta b$$

$$iv) a\alpha(b\beta c)b = (a\alpha b)\beta c \text{ for all } a, b, c \in \mathbb{Z}_8 \text{ and for all } \alpha, \beta \in \Gamma = \{2, 4, 6\}.$$

Hence  $\mathbb{Z}_8$  is a  $\Gamma$ - semiring.

Let  $N = \mathbb{Z}_8$  and  $\psi : N \rightarrow P(\mathbb{Z}_8)$  be a set valued function defined by

$$\psi(y) = \{s \in \mathbb{Z}_8 : R(y, \alpha, s) \leftrightarrow, (y, \alpha, s) \in \{0, 4, 6\}, \forall \alpha \in \Gamma\}$$

for all  $y \in N = \mathbb{Z}_8$ . Then

$$\psi(0) = \psi(2) = \psi(4) = \psi(6) = \mathbb{Z}_8$$

$$\psi(1) = \psi(3) = \psi(5) = \psi(7) = \{0, 2, 4, 6\}$$

are sub- $\Gamma$ -semirings of  $\mathbb{Z}_8$ . Hence  $(\psi, N)$  is a soft- $\Gamma$ - semiring over  $\mathbb{Z}_8$ .

**Proposition 3.4** *Let  $(\rho, W, \Gamma)$  and  $(\sigma, W, \Gamma)$  be soft  $\Gamma$  semirings over  $\Gamma$ -semiring  $S$ . The restricted intersection  $(\rho, W, \Gamma) \widetilde{\cap}_{\mathfrak{R}} (\sigma, W, \Gamma)$  is a soft  $\Gamma$  semiring over  $S$  if it is non-null.*

**Proof.** By Definition 2.3 (i), we have that  $(\rho, W, \Gamma) \widetilde{\cap}_{\mathfrak{R}} (\sigma, W, \Gamma) = (\psi, W, \Gamma)$  where  $\psi(\omega) = \rho(\omega) \cap \sigma(\omega)$  for all  $\omega \in W$ . We assume that  $(\psi, W, \Gamma)$  is a non-null soft set over  $S$ . If  $\omega \in \text{Supp}(\psi, W, \Gamma)$ , then  $\psi(\omega) = \rho(\omega) \cap \sigma(\omega) \neq \emptyset$ . We know that  $(\rho, W, \Gamma)$  and  $(\sigma, W, \Gamma)$  are both soft  $\Gamma$  semirings over  $S$ , and so, the nonempty sets  $\rho(\omega)$  and  $\sigma(\omega)$  are both sub  $\Gamma$  semiring of  $S$  (From definition 3.2). Thus,  $\psi(\omega)$  is a sub  $\Gamma$ -semiring of  $S$  for all  $\omega \in \text{Supp}(\psi, W, \Gamma)$ . In this position,  $(\psi, W, \Gamma) = (\rho, W, \Gamma) \widetilde{\cap}_{\mathfrak{R}} (\sigma, W, \Gamma)$  is a soft  $\Gamma$  semiring over  $S$  ■

**Corollary 3.5** *Let  $\{(\rho_i, W_i, \Gamma) : i \in I\}$  be a nonempty family of soft  $\Gamma$ -semiring over  $S$ . Then the restricted intersection  $(\widetilde{\cap}_{\mathfrak{R}})_{i \in I} (\rho_i, W_i, \Gamma)$  is a soft  $\Gamma$ -semiring over  $S$  if it is non-null.*

**Proof.** Straight forward ■

**Theorem 3.6** *Let  $(\rho_i, W_i, \Gamma)_{i \in I}$  be a nonempty family of soft- $\Gamma$ -semirings over  $S$ . Then the restricted intersection  $(\widetilde{\cap}_{\mathfrak{R}})_{i \in I} (\rho_i, W_i, \Gamma)$  is a soft  $\Gamma$ -semiring over  $S$  if it is non-null.*

**Proof.** From definition 2.3(ii), we have that  $(\widetilde{\cap}_{\mathfrak{R}})_{i \in I} (\rho_i, W_i, \Gamma) = (\psi, Y, \Gamma)$ , where  $Y = \bigcap_{i \in I} W_i \neq \emptyset$ , and  $\psi(y) = \bigcap_{i \in I} \rho_i(y)$  for every  $y \in Y$ .

We assume that  $(\psi, Y, \Gamma)$  is non-null. Let  $y \in \text{Supp}(\psi, Y, \Gamma)$ . Then  $\psi(y) \neq \emptyset$  and so we have  $\rho_i(y) \neq \emptyset$  for every  $i \in I$ . From the hypothesis, we know that  $\{(\rho_i, W_i, \Gamma) : i \in I\}$  is a nonempty family of soft- $\Gamma$ -semiring over  $S$ , by definition 3.2  $\rho_i(y)$  is a sub- $\Gamma$ -semiring of  $S$ , that is,  $\psi(y)$  is a sub- $\Gamma$ -semiring of  $S$  for all  $y \in \text{Supp}(\psi, Y, \Gamma)$  and so  $(\psi, Y, \Gamma)$  is a soft  $\Gamma$  semiring over  $S$ . ■

**Theorem 3.7** *Let  $\{(\rho_i, W_i, \Gamma) : i \in I\}$  be a nonempty family of soft  $\Gamma$ -semiring over  $S$ . Then the extended intersection  $(\widetilde{\cap}_{\mathcal{E}})_{i \in I} (\rho_i, W_i, \Gamma)$  is a soft  $\Gamma$ -semirings over  $S$ .*

**Proof.** From definition 2.4 (ii), we have that  $(\widetilde{\cap}_{\mathcal{E}})_{i \in I} (\rho_i, W_i, \Gamma) = (\psi, Y, \Gamma)$  where  $Y = \bigcup_{i \in I} W_i$ , and  $\psi(y) = \bigcap_{i \in I} \rho_i(y)$  for all  $y \in Y$ .

Assume that  $y \in \text{Supp}(\psi, Y, \Gamma)$ . Then  $\psi(y) \neq \emptyset$  and so we have  $\rho_i(y) \neq \emptyset$  for every  $i \in I$ . Because of the fact that  $\{(\rho_i, W_i, \Gamma) : i \in I\}$  is a soft  $\Gamma$ -semiring over  $S$  for every  $i \in I$ , we have that  $\rho_i(y)$  is a sub  $\Gamma$ -semiring over  $S$  for every  $i \in I$ . It follows that  $\psi(y) = \bigcap_{i \in I} \rho_i(y)$  is a sub- $\Gamma$ -semiring over  $S$  for every  $y \in \text{Supp}(\psi, Y, \Gamma)$ . Thus,  $(\widetilde{\cap}_{\mathcal{E}})_{i \in I} (\rho_i, W_i, \Gamma)$  is a soft- $\Gamma$ -semiring over  $S$ . ■

**Theorem 3.8** *Let  $\{(\rho_i, W_i, \Gamma) : i \in I\}$  be a nonempty family of soft  $\Gamma$ -semirings over  $S$ . If  $\rho_i(y_i) \subseteq \rho_j(y_j)$  or  $\rho_j(y_j) \subseteq \rho_i(y_i)$  for all  $i, j \in I, y_i \in W_i$  then the restricted union  $(\widetilde{\cup}_{\mathfrak{R}})_{i \in I} (\rho_i, W_i)$  is a soft- $\Gamma$ -semiring over  $S$ .*



**Proof.** Using definition 2.5 (ii), we have that  $(\widetilde{\cup}_{\mathcal{R}})_{i \in I}(\rho_i, W_i, \Gamma) = (\psi, Y, \Gamma)$  where  $Y = \bigcap_{i \in I} W_i$ , and  $\psi(y) = \bigcup_{i \in I} \rho_i(y)$  for all  $y \in Y$ . Assume that  $y \in \text{Supp}(\psi, Y, \Gamma)$ . Then  $\psi(y) \neq \emptyset$  and so we have  $\rho_{i_0}(y) \neq \emptyset$  for some  $i_0 \in I(y)$ . By assumption,  $\bigcup_{i \in I} \rho_i(y)$  is a sub  $\Gamma$ -semiring of  $S$  for every  $y \in \text{Supp}(\psi, Y, \Gamma)$ . Hence,  $(\widetilde{\cup}_{\mathcal{R}})_{i \in I}(\rho_i, W_i, \Gamma)$  is a soft  $\Gamma$ -semiring over  $S$ . ■

**Theorem 3.9** *Let  $\{(\rho_i, W_i, \Gamma) : i \in I\}$  be a nonempty family of soft  $\Gamma$ -semiring over  $S$ . Let  $W_i$  and  $W_j$  be members of the family  $\{W_i : i \in I\}$  such that  $W_i \cap W_j = \emptyset$  for  $i \neq j$ . Then  $(\widetilde{\cup}_{\mathcal{E}})_{i \in I}(\rho_i, W_i, \Gamma)$  is a soft  $\Gamma$ -semiring over  $S$ .*

**Proof.** From definition 2.6 (ii) we have that where  $(\widetilde{\cup}_{\mathcal{E}})_{i \in I}(\rho_i, W_i, \Gamma) = (\psi, Y, \Gamma)$   $\psi(y) = \bigcap_{i \in I} \rho_i(y)$  for all  $y \in Y$ . Note first that  $(\psi, Y)$  is non-null owing to the fact that  $\text{Supp}(\psi, Y, \Gamma) = \bigcup_{i \in I} \text{Supp}(\rho_i, W_i, \Gamma)$ . Suppose that  $y \in \text{Supp}(\psi, Y, \Gamma)$ . Then  $\psi(y) \neq \emptyset$  so we have  $\rho_{i_0} \neq \emptyset$  for some  $i_0 \in I(y)$ . From the hypothesis  $\{W_i : i \in I\}$  are pairwise disjoint, we follow that  $\varphi(y) = \rho_{i_0}(y)$ . On the other hand  $\rho_{i_0}(y)$  is a soft  $\Gamma$ -semiring over  $S$ , we conclude that  $(\psi, Y)$  is a soft  $\Gamma$ -semiring over  $S$  for all  $y \in (\psi, Y, \Gamma)$ . Consequently  $(\widetilde{\cup}_{\mathcal{E}})_{i \in I}(\rho_i, W_i, \Gamma) = (\psi, Y, \Gamma)$  is a soft  $\Gamma$ -semiring over  $S$ . ■

**Theorem 3.10** *If  $(\rho, W, \Gamma)$  and  $(\sigma, Y, \Gamma)$  be two soft  $\Gamma$ -semirings over  $\Gamma$ -semiring  $S$ , then  $(\rho, W, \Gamma) \widetilde{\wedge} (\sigma, Y, \Gamma)$  is a soft  $\Gamma$ -semiring over  $S$  if it is non-null.*

**Proof.** Using definition 2.7 (i), we have that  $(\rho, W, \Gamma) \widetilde{\wedge}_{\mathcal{E}} (\sigma, Y, \Gamma) = (\psi, Z, \Gamma)$ , where  $Z = W \times \Gamma \times Y$  and  $\psi(\omega, \alpha, y) = \rho(\omega) \cap \sigma(y)$  for all  $(\omega, \alpha, y) \in Z = W \times \Gamma \times Y$ . Then by the hypothesis,  $(\psi, Z, \Gamma)$  is a nonnull soft set over  $\Gamma$ -semiring  $S$ . Since  $(\psi, Z, \Gamma)$  is a nonnull,  $\text{Supp}(\psi, Z, \Gamma) \neq \emptyset$  and so, for  $(\omega, \alpha, y) \in \text{Supp}(\psi, Z, \Gamma)$ ,  $\psi(\omega, \alpha, y) = \rho(\omega) \cap \sigma(y) \neq \emptyset$ . We assume that  $t_1, t_2 \in \rho(\omega) \cap \sigma(y)$ . In this position

- i) If  $t_1, t_2 \in \rho(\omega) = \{y : R(\omega, \alpha_1, y), \forall \alpha_1 \in \Gamma\}$  we have that  $\omega \alpha_1 t_1 \in W, \omega \alpha_1 t_2 \in W$ . This implies  $\omega \alpha_1 (t_1 + t_2) \in W$  and
- ii)  $t_1, t_2 \in \sigma(y) = \{y' : R(y, \alpha_2, y'), \forall \alpha_2 \in \Gamma\}$  we have that  $y \alpha_2 t_1 \in Y, y \alpha_2 t_2 \in Y$ . This implies  $y \alpha_2 (t_1 + t_2) \in Y$ .

Hence  $\rho(x) \cap \sigma(y)$  is a sub- $\Gamma$  semiring. By definition of soft  $\Gamma$  semiring,  $(\rho, W, \Gamma)$  and  $(\sigma, Y, \Gamma)$  are both soft  $\Gamma$  semirings over  $S$ .  $\rho(x)$  and  $\sigma(y)$  are also sub- $\Gamma$  semiring of  $S$ . Furthermore  $\psi(\omega, \alpha, y) = \rho(\omega) \cap \sigma(y)$  is a sub  $\Gamma$  semiring of  $S$  for all  $(\omega, \alpha, y) \in (\psi, Z, \Gamma) = (\rho, W, \Gamma) \widetilde{\wedge} (\sigma, Y, \Gamma)$  is a soft  $\Gamma$  semiring over  $S$  required.

■

**Theorem 3.11** *Let  $\{(\rho_i, W_i, \Gamma) : i \in I\}$  be a nonempty family of soft  $\Gamma$ -semiring over  $S$ . Then  $\widetilde{\wedge}_{i \in I}(\rho_i, W_i, \Gamma)$  is a soft  $\Gamma$ -semiring over  $S$  if it is non-null.*

**Proof.** By taking into account to the definition 2.7 (ii) we write  $\tilde{\wedge}_{i \in I} (\rho_i, W_i, \Gamma) = (\psi, Y, \Gamma)$ , where  $Y = \prod_{i \in I} W_i$ , and  $\psi(y) = \bigcap_{i \in I} \rho_i(y)$  for all  $y = (y_i)_{i \in I} \in Y$ .

Suppose that  $(\psi, Y, \Gamma)$  is non-null. If  $y = (y_i)_{i \in I} \in \text{Supp}(\psi, Y, \Gamma)$ , then  $\psi(y) \neq \emptyset$ . Since  $(\rho_i, W_i, \Gamma)$  is a soft  $\Gamma$ -semiring over  $S$  for all  $i \in I$  members of nonempty family  $((\rho_i, W_i, \Gamma) : i \in I)$  such that  $\rho_i(y_i)$  is a sub  $\Gamma$ -semiring of  $S$ . Hence  $\psi(y)$  is a sub  $\Gamma$ -semiring of  $S$  for all  $y \in \text{Supp}(\psi, Y, \Gamma)$ , and so  $\tilde{\wedge}_{i \in I} (\rho_i, W_i, \Gamma) = (\psi, Y, \Gamma)$  is soft  $\Gamma$ -semiring over  $S$ . ■

**Theorem 3.12** *Let  $\{(\rho_i, W_i, \Gamma) : i \in I\}$  be a nonempty family of soft  $\Gamma$ -semiring over  $S$ . If  $\rho_i(y_i) \subseteq \rho_j(y_j)$  or  $\rho_j(y_j) \subseteq \rho_i(y_i)$  for all  $i, j \in I, y_i \in W_i$ , the  $\vee$ -union  $\tilde{\vee}_{i \in I} (\rho_i, W_i, \Gamma)$  is a soft  $\Gamma$ -semiring over  $S$ .*

**Proof.** Using the definition 2.8 (ii), we have that  $\tilde{\vee}_{i \in I} (\rho_i, W_i, \Gamma) = (\psi, Y, \Gamma)$  where  $Y = \prod_{i \in I} A_i$ , and  $\psi(y) = \bigcup_{i \in I} \rho_i(y)$  for all  $y = (y_i)_{i \in I} \in Y$ .

Assume that  $y = (y_i)_{i \in I} \in \text{Supp}(\psi, Y, \Gamma)$ . Then  $\psi(y) \neq \emptyset$  and so we have that  $\rho_{i_0}(y) \neq \emptyset$  for some  $i_0 \in I$ . By assumption,  $\bigcup_{i \in I} \rho_i(y)$  is a soft  $\Gamma$ -semiring of  $S$  for all  $y = (y_i)_{i \in I} \in \text{Supp}(\psi, Y, \Gamma)$ . Consequently  $\tilde{\vee}_{i \in I} (\rho_i, W_i, \Gamma) = (\psi, Y, \Gamma)$  is a soft- $\Gamma$ -semiring over  $S$  ■

**Theorem 3.13** *Let  $\{(\rho_i, W_i, \Gamma) : i \in I\}$  be a nonempty family of soft  $\Gamma$ -semirings over  $S_i$ . Then  $\tilde{\prod}_{i \in I} (\rho_i, W_i, \Gamma)$  is a soft  $\Gamma$ -semiring over  $\prod_{i \in I} S_i$ .*

**Proof.** By definition 2.10 we write  $\tilde{\prod}_{i \in I} (\rho_i, W_i, \Gamma) = (\psi, Y, \Gamma)$ , where  $Y = \prod_{i \in I} W_i$ , and  $\psi(y) = \prod_{i \in I} \rho_i(y)$  for all  $y = (y_i)_{i \in I} \in Y$ .

Let  $y = (y_i)_{i \in I} \in \text{Supp}(\psi, Y, \Gamma)$ . Then  $\psi(y) \neq \emptyset$ , and so we have  $\rho_i(y_i) \neq \emptyset$  for all  $i \in I$ . By taking into account,  $\{(\rho_i, W_i, \Gamma) : i \in I\}$  is a soft  $\Gamma$ -semiring over  $S_i$  for all  $i \in I$ , it follows that  $\prod_{i \in I} \rho_i(y_i)$  is a soft- $\Gamma$ -semiring of  $\prod_{i \in I} S_i$  for all  $y = (y_i)_{i \in I} \in \text{Supp}(\psi, Y, \Gamma)$ . Hence  $\tilde{\prod}_{i \in I} (\rho_i, W_i, \Gamma)$  is a soft  $\Gamma$ -semiring over  $\prod_{i \in I} S_i$  ■

**Definition 3.14** *Let  $(\rho, W, \Gamma)$  be soft  $\Gamma$ -semiring over  $S$ .*

- i)  $(\rho, W, \Gamma)$  is called the trivial soft  $\Gamma$ -semiring over  $S$  if  $\rho(\omega) = \{0\}$  for all  $\omega \in W$
- ii)  $(\rho, W, \Gamma)$  is called the whole soft  $\Gamma$ -semiring over  $S$  if  $\rho(\omega) = S$  for all  $\omega \in W$

**Definition 3.15** *Let  $S$  and  $S'$  be two  $\Gamma$ -semiring and  $f : S \rightarrow S'$  a mapping of  $\Gamma$ -semiring. If  $(\rho, W)$  and  $(\sigma, Y)$  are soft sets over  $S$  and  $S'$  respectively, then*

- i)  $(f(\rho), W)$  is a soft set over  $S'$  where

$$f(\rho) : W \rightarrow P(S')$$

$$f(\rho)(\omega) = f(\rho(w))$$

for all  $\omega \in W$ .

ii)  $(f^{-1}(\sigma), Y)$  is a soft set over  $S$  where

$$f^{-1}(\sigma) : Y \rightarrow P(S)$$

$$f^{-1}(\sigma)(y) = f^{-1}(\sigma(y))$$

for all  $y \in Y$ .

**Lemma 3.16** *Let  $f : S \rightarrow S'$  be an onto homomorphism of  $\Gamma$ -semiring. The following statements can be given.*

- i)  $(\rho, W, \Gamma)$  be soft  $\Gamma$ -semiring over  $S$ , then  $(f(\rho), W, \Gamma)$  is a soft  $\Gamma$ -semiring over  $S'$
- ii)  $(\sigma, Y, \Gamma)$  be soft  $\Gamma$ -semiring over  $S$ , then  $(f^{-1}(\sigma), Y, \Gamma)$  is a soft  $\Gamma$ -semiring over  $S$ .

**Proof.**

- i) Since  $(\rho, W, \Gamma)$  is a soft  $\Gamma$ -semiring over  $S$ , it is clear that  $(f(\rho), W)$  is a non-null soft set over  $S'$ . For every  $y \in \text{Supp}(f(\rho), W, \Gamma)$  we have  $f(\rho)(y) = f(\rho(y)) \neq \emptyset$ . Hence  $f(\rho(y))$  which is the onto homomorphic image of  $\Gamma$ -semiring  $\rho(y)$  is a  $\Gamma$ -semiring of  $S'$  for all  $y \in \text{Supp}(\rho(f), W, \Gamma)$ . That is  $(f(\rho), W, \Gamma)$  is a soft  $\Gamma$ -semiring of  $S'$ .
- ii) It is easy to see that  $\text{Supp}(f^{-1}(\sigma), Y, \Gamma) \subseteq \text{Supp}(\sigma, Y, \Gamma)$ . By this way let  $y \in \text{Supp}(f^{-1}(\sigma), Y, \Gamma)$ . Then  $\sigma(y) \neq \emptyset$ . Hence  $f^{-1}(\sigma(y))$  which is homomorphic inverse image of  $\Gamma$ -semiring  $\sigma(y)$ , is a soft  $\Gamma$ -semiring over  $S$  for all  $y \in Y$ .

■

**Theorem 3.17** *Let  $f : S \rightarrow S'$  be a homomorphism of  $\Gamma$ -semiring. Let  $(\rho, W, \Gamma)$  and  $(\sigma, Y, \Gamma)$  be two soft  $\Gamma$ -semiring over  $S$  and  $S'$ , respectively. Then the following are given.*

- i) If  $\rho(\omega) = \ker(f)$  for all  $\omega \in W$ , then  $(f(\rho), W, \Gamma)$  is the trivial soft  $\Gamma$ -semiring over  $S'$ .
- ii) If  $f$  is onto and  $(\rho, W)$  is whole, then  $(f(\rho), W, \Gamma)$  is the whole soft  $\Gamma$ -semiring over  $S'$ .
- iii) If  $\sigma(y) = f(S)$  for all  $y \in Y$ , then  $(f^{-1}(\sigma), Y, \Gamma)$  is the whole soft  $\Gamma$ -semiring over  $S$ .
- iv) If  $f$  is injective and  $(\sigma, Y)$  is trivial, then  $(f^{-1}(\sigma), Y, \Gamma)$  is the trivial soft  $\Gamma$ -semiring over  $S$ .

**Proof.**

- i) By using  $\rho(\omega) = \ker(f)$  for all  $y \in W$ . Then  $f(\rho)(\omega) = f(\rho(\omega)) = \{0_{S'}\}$  for all  $\omega \in W$ . Hence  $(f(\rho), W, \Gamma)$  is soft  $\Gamma$ -semiring over  $S'$  by Lemma 3.16 and Definition 3.14.
- ii) Suppose that  $f$  is onto and  $(\rho, W)$  is whole. Then  $\rho(\omega) = S$  for all  $\omega \in W$ , and so  $f(\rho)(\omega) = f(\rho(\omega)) = f(S) = S'$  for all  $\omega \in W$ . It follows from Lemma 3.16 and Definition 3.14 that  $(f(\rho), W)$  is the whole soft  $\Gamma$ -semiring over  $S'$ .
- iii) If we use hypothesis  $\sigma(y) = f(S)$  for all  $y \in Y$ , we can write  $f^{-1}(\sigma)(y) = f^{-1}(\sigma(y)) = f^{-1}(f(S)) = S$  for all  $y \in Y$ . It is clear that,  $(f^{-1}(\sigma), B, \Gamma)$  is the whole soft  $\Gamma$ -semiring over  $S$  by Lemma 3.16 and Definition 3.14.
- iv) Suppose that  $f$  is injective and  $(\sigma, Y)$  is trivial. Then,  $\sigma(y) = \{0\}$  for all  $y \in Y$ , so  $f^{-1}(\sigma)(y) = f^{-1}(\sigma(y)) = f^{-1}(\{0\}) = \ker f = \{0_S\}$  for all  $y \in Y$ . It follows from Lemma 3.16 and Definition 3.14 that  $(f^{-1}(\sigma), Y, \Gamma)$  is the trivial soft  $\Gamma$ -semiring over  $S$ .

■

## 4 Soft Sub $\Gamma$ -Semiring

**Definition 4.1** Let  $(\rho, W, \Gamma)$  and  $(\sigma, Y, \Gamma)$  be two soft  $\Gamma$ -semirings over  $S$ . Then the soft  $\Gamma$ -semiring is called a soft sub  $\Gamma$ -semiring of  $(\rho, W, \Gamma)$ , denoted by  $(\sigma, Y, \Gamma) \subset_{\Gamma_s} (\rho, W, \Gamma)$ , if it satisfies the following conditions

- i)  $Y \subseteq W$ ,
  - ii)  $\sigma(y)$  is a sub  $\Gamma$ -Semiring of  $\rho(y)$  for all  $y \in \text{Supp}(\sigma, Y, \Gamma)$ .
- From the above definition, it is easily deduced that if  $(\sigma, Y, \Gamma)$  is a soft sub  $\Gamma$ -Semiring of  $(\rho, W, \Gamma)$ , then  $\text{Supp}(\sigma, Y, \Gamma) \subset \text{Supp}(\rho, W, \Gamma)$ .

**Theorem 4.2** Let  $(\rho, W, \Gamma)$  and  $(\sigma, Y, \Gamma)$  be two soft  $\Gamma$ -semirings over  $S$  and  $(\rho, W, \Gamma) \supseteq (\sigma, Y, \Gamma)$ . Then  $(\sigma, Y, \Gamma) \subset_{\Gamma_s} (\rho, W, \Gamma)$ ,

**Proof.** Straightforward. ■

**Theorem 4.3** Let  $(\rho, W, \Gamma)$  and  $(\sigma, Y, \Gamma)$  be two soft  $\Gamma$ -semirings over  $S$  and  $(\rho, W, \Gamma) \widetilde{\cap} (\sigma, Y, \Gamma)$  is a soft sub  $\Gamma$  semiring of both  $(\rho, W, \Gamma)$  and  $(\sigma, Y, \Gamma)$  if it is non-null.

**Proof.** Straightforward. ■

**Theorem 4.4** Let  $(\rho, W, \Gamma)$  be soft  $\Gamma$ -semiring over  $S$  and  $\{(\psi_i, W_i, \Gamma) : i \in I\}$  be nonempty family of soft sub  $\Gamma$ -semirings of  $(\rho, W, \Gamma)$ . Then the restricted intersection  $(\widetilde{\cap}_{\mathfrak{R}})_{i \in I} (\psi_i, W_i, \Gamma)$  is a soft sub  $\Gamma$ -semiring of  $(\rho, W, \Gamma)$  if it is non-null.

**Proof.** Similar to the proof of Theorem 3.6. ■

**Corollary 4.5** *Let  $(\rho, W, \Gamma)$  be soft  $\Gamma$ -semiring over  $S$  and  $\{(\psi_i, W_i, \Gamma) : i \in I\}$  be nonempty family of soft sub  $\Gamma$ -semirings of  $(\rho, W, \Gamma)$ . Then  $(\bigcap_{i \in I} (\psi_i, W_i, \Gamma))$  is a soft sub  $\Gamma$ -semiring of  $(\rho, W, \Gamma)$  if it is non-null.*

**Proof.** Straightforward. ■

**Theorem 4.6** *Let  $(\rho, W, \Gamma)$  be soft  $\Gamma$ -semiring over  $S$  and  $\{(\psi_i, W_i, \Gamma) : i \in I\}$  be nonempty family of soft sub  $\Gamma$ -semirings of  $(\rho, W, \Gamma)$ . Then the extended intersection  $(\bigcap_E)_{i \in I} (\psi_i, W_i, \Gamma)$  is a soft sub  $\Gamma$ -semiring of  $(\rho, W, \Gamma)$ .*

**Proof.** Similar to the proof of Theorem 3.7. ■

**Theorem 4.7** *Let  $(\rho, W, \Gamma)$  be soft  $\Gamma$ -semiring over  $S$  and  $\{(\psi_i, W_i, \Gamma) : i \in I\}$  be nonempty family of soft sub  $\Gamma$ -semirings of  $(\rho, W, \Gamma)$ . If  $\psi_i(y_i) \subseteq \psi_j(y_j)$  or  $\psi_j(y_j) \subseteq \psi_i(y_i)$  for all  $i, j \in I, y_i \in W_i$ , then the restricted union  $(\bigcup_{\mathcal{R}})_{i \in I} (\psi_i, W_i, \Gamma)$  is a soft sub  $\Gamma$ -semiring of  $(\rho, W, \Gamma)$ .*

**Proof.** By the aid of the definition 2.6 (ii), we write  $(\bigcup_{\mathcal{E}})_{i \in I} (\psi_i, W_i, \Gamma) = (\psi, Y, \Gamma)$ , where  $Y = \bigcup_{i \in I} W_i$ , and  $\psi(y) = \bigcup_{i \in I} \psi_i(y)$  for all  $y \in Y$ .

Let  $y \in \text{Supp}(\psi, Y, \Gamma)$ . Then  $\psi(y) \neq \emptyset$ , and so we have  $\psi_{i_0}(y_{i_0}) \neq \emptyset$  for some  $i_0 \in I$ . From the hypothesis, we know that  $\psi_i(y_i) \subseteq \psi_j(y_j)$  or  $\psi_j(y_j) \subseteq \psi_i(y_i)$  for all  $i, j \in I, y_i \in W_i$ , clearly  $\bigcup_{i \in I} \psi_i(y)$  is a sub  $\Gamma$ -semiring of  $\rho(y)$  for all  $y \in \text{Supp}(\psi, Y, \Gamma)$ . Thus  $(\bigcup_{\mathcal{R}})_{i \in I} (\psi_i, W_i, \Gamma) = (\psi, Y, \Gamma)$  is a soft sub  $\Gamma$ -semiring of  $(\rho, W, \Gamma)$ . ■

**Theorem 4.8** *Let  $(\rho, W, \Gamma)$  be soft  $\Gamma$ -semiring over  $S$  and  $\{(\psi_i, W_i, \Gamma) : i \in I\}$  be nonempty family of soft sub  $\Gamma$ -semirings of  $(\rho, W, \Gamma)$ . If  $\psi_i(y_i) \subseteq \psi_j(y_j)$  or  $\psi_j(y_j) \subseteq \psi_i(y_i)$  for all  $i, j \in I, y_i \in W_i$ , then  $\vee$  union  $\bigvee_{i \in I} (\psi_i, W_i, \Gamma)$  is a soft sub  $\Gamma$ -semiring of  $(\rho, W, \Gamma)$ .*

**Proof.** Similar to the proof of Theorem 3.12. ■

**Theorem 4.9** *Let  $(\rho, W, \Gamma)$  be a soft  $\Gamma$ -semiring over  $S$  and  $\{(\psi_i, W_i, \Gamma) : i \in I\}$  be nonempty family of soft sub  $\Gamma$ -semirings of  $(\rho, W, \Gamma)$ . Then the  $\wedge$  intersection  $\bigwedge_{i \in I} (\psi_i, W_i, \Gamma)$  is a soft sub  $\Gamma$ -semiring of  $(\rho, W, \Gamma)$ .*

**Proof.** Similar to the proof of Theorem 3.11. ■

**Theorem 4.10** *Let  $(\rho, W, \Gamma)$  be soft  $\Gamma$ -semiring over  $S$  and  $\{(\psi_i, W_i, \Gamma) : i \in I\}$  be nonempty family of soft sub  $\Gamma$ -semirings of  $(\rho, W, \Gamma)$ . Then the cartesian product of the family  $\prod_{i \in I} (\psi_i, W_i, \Gamma)$  is a soft sub  $\Gamma$ -semiring of  $(\rho, W, \Gamma)$ .*

**Proof.** By Definition 2.10, we can write  $\prod_{i \in I} (\psi_i, W_i, \Gamma) = (\psi, Y, \Gamma)$  where  $Y = \prod_{i \in I} W_i$  and  $\psi(y) = \prod_{i \in I} \psi_i(y_i)$  for all  $y = (y_i)_{i \in I} \in Y$ . Let  $y = (y_i)_{i \in I} \in \text{Supp}(\psi, Y, \Gamma)$ . Then  $\psi(y) \neq \emptyset$  and so we have  $\psi_i(y_i) \neq \emptyset$  for all  $i \in I$ . In as much as  $\{(\psi_i, W_i, \Gamma) : i \in I\}$  is a soft sub  $\Gamma$ -semiring of  $(\rho, W, \Gamma)$ , we have that  $\psi_i(y_i)$  is a sub  $\Gamma$ -semiring of  $\rho(y_i)$ . It follows that, we obtain  $\prod_{i \in I} \psi_i(y_i)$  for all  $y = (y_i)_{i \in I} \in \text{Supp}(\psi, Y, \Gamma)$ . Hence, the cartesian product of the family  $\prod_{i \in I} (\rho_i, W_i, \Gamma)$  is a soft sub  $\Gamma$ -semiring of  $(\rho, W, \Gamma)$ . ■

**Theorem 4.11** *Let  $f : S \rightarrow S'$  be a homomorphism of  $\Gamma$ -semirings and  $(\rho, W, \Gamma)$  and  $(\sigma, Y, \Gamma)$  two soft  $\Gamma$ -semirings over  $S$ . If  $(\sigma, Y, \Gamma) \subset_{\Gamma_S} (\rho, W, \Gamma)$  then  $(f(\sigma), Y, \Gamma) \subset_{\Gamma_S} (f(\rho), W, \Gamma)$ .*

**Proof.** Suppose that  $y \in \text{Supp}(\sigma, Y, \Gamma)$ . Then  $y \in \text{Supp}(\rho, W, \Gamma)$ . By definition 4.1, we know that  $Y \subseteq W$  and  $\sigma(y)$  is a sub  $\Gamma$ -semiring of  $\rho(y)$  for all  $y \in \text{Supp}(\sigma, Y, \Gamma)$ . From the expression hypothesis  $f$  is a homomorphism,  $f(\sigma)(y) = f(\sigma(y))$  is a sub  $\Gamma$ -semiring of  $f(\rho)(y) = f(\rho(y))$  and therefore  $(f(\sigma), Y, \Gamma) \subset_{\Gamma_S} (f(\rho), W, \Gamma)$ . ■

**Theorem 4.12** *Let  $f : S \rightarrow S'$  be a homomorphism of  $\Gamma$ -semiring and  $(\rho, W, \Gamma)$ ,  $(\sigma, Y, \Gamma)$  two soft  $\Gamma$ -semirings over  $S$ . If  $(\sigma, Y, \Gamma) \subset_{\Gamma_S} (\rho, W, \Gamma)$  then  $(f^{-1}(\sigma), Y, \Gamma) \subset_{\Gamma_S} (f^{-1}(\rho), W, \Gamma)$ .*

**Proof.** Let  $y \in \text{Supp}(f^{-1}(\sigma), Y, \Gamma)$ .  $Y \subseteq W$  and  $\sigma(y)$  is a sub  $\Gamma$ -semiring of  $\rho(y)$  for all  $y \in Y$ . Since  $f$  is a homomorphism,  $f^{-1}(\sigma)(y) = f^{-1}(\sigma(y))$  is a sub  $\Gamma$ -semiring of  $f^{-1}(\sigma(y)) = f(\sigma)(y)$  for all  $y \in \text{Supp}(f^{-1}(\sigma), Y, \Gamma)$ . Hence  $(f^{-1}(\sigma), Y, \Gamma) \subset_{\Gamma_S} (f^{-1}(\rho), W, \Gamma)$  ■

**Definition 4.13** *Let  $(\rho, W, \Gamma)$  and  $(\sigma, Y, \Gamma)$  be two soft  $\Gamma$ -semiring over  $S$  and  $S'$ , respectively. Let  $f : S \rightarrow S'$  and  $g : W \rightarrow Y$  be two functions. The following conditions:*

i)  *$f$  is an epimorphism of  $\Gamma$ -semiring*

ii)  *$g$  is and surjective mapping.*

ii)  *$f(\rho(y)) = \sigma(\rho(y))$  for all  $y \in W$ .*

*where satisfied by the pair  $(f, g)$ , then  $(f, g)$  is called soft  $\Gamma$ - semiring homomorphism.*

*If there exists a soft Gamma-semiring homomorphism between  $(\rho, W, \Gamma)$  and  $(\sigma, Y, \Gamma)$ , we say that  $(\rho, W, \Gamma)$  is soft homomorphic to  $(\sigma, Y, \Gamma)$ , and is denoted by  $(\rho, W, \Gamma) \sim_{\Gamma_S} (\sigma, Y, \Gamma)$  is soft isomorphic to  $(G, B, \Gamma)$ , which is denoted by  $(\rho, W, \Gamma) \simeq_{\Gamma_S} (\sigma, Y, \Gamma)$ .*

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